Simplify the following expressions completely. Show proper reasoning to justify your answer.   SCORE: _____ / 12 PTS
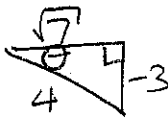
[a] $\cot(\arcsin(-\frac{3}{4}))$

$\underbrace{\qquad}_{\theta}$

$\frac{\sqrt{7}}{\theta}\bigg/\!\!\!\!\!\Big|{}^{4}_{-3}$

4

$\sin\theta = -\frac{3}{4}$

$\cot\theta = -\frac{\sqrt{7}}{3}$

[b] $\csc(\tan^{-1} 2x)$

$\underbrace{\qquad}_{\theta}$

$\sqrt{1+4x^2}\,\Big|\!\!\!\!\diagdown\,{}^{2x}_{1}$
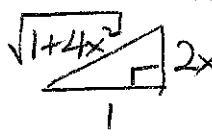
$\tan\theta = 2x$

$\csc\theta = \frac{\sqrt{1+4x^2}}{2x}$

Graph 2 periods of the function $y = -3\cos(\frac{3\pi}{2}x + \frac{13\pi}{6}) + 2$.   SCORE: _____ / 16 PTS

Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete periods, starting at the phase shift.

Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.

MIDDLE = 2

AMPLITUDE = $|-3| = 3$

MAX = $2+3 = 5$

MIN = $2-3 = -1$

$-\cos$ $\frown$

PERIOD = $\frac{2\pi}{\frac{3\pi}{2}} = 2\pi \cdot \frac{2}{3\pi} = \frac{4}{3}$

$\frac{1}{4}$ PERIOD = $\frac{1}{3}$

PHASE SHIFT $\frac{3\pi}{2}x + \frac{13\pi}{6} = 0$

$x = -\frac{13\pi}{6} \cdot \frac{2}{3\pi} = \frac{-13}{9}$

POINTS: $(\ \frac{-13}{9}\ ,\ -1\ )$

$(\ \frac{-10}{9}\ ,\ 2\ )$

$(\ \frac{-7}{9}\ ,\ 5\ )$

$(\ \frac{-4}{9}\ ,\ 2\ )$

$(\ \frac{-1}{9}\ ,\ -1\ )$

$(\ \frac{2}{9}\ ,\ 2\ )$

$(\ \frac{5}{9}\ ,\ 5\ )$

$(\ \frac{8}{9}\ ,\ 2\ )$
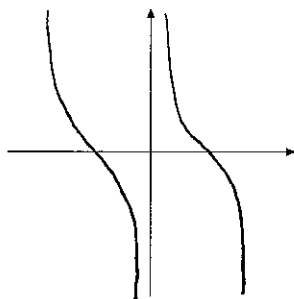
$(\ \frac{11}{9}\ ,\ -1\ )$
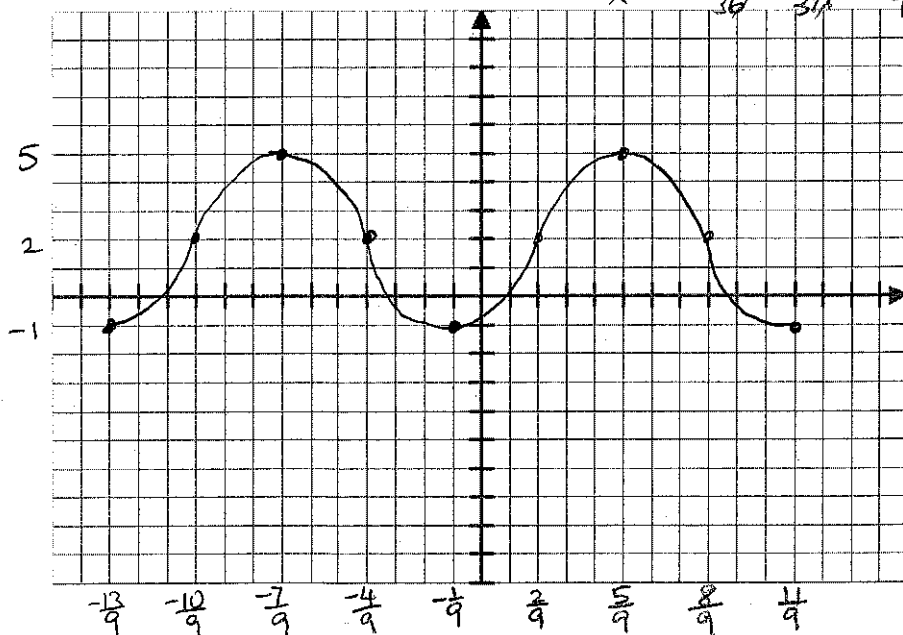
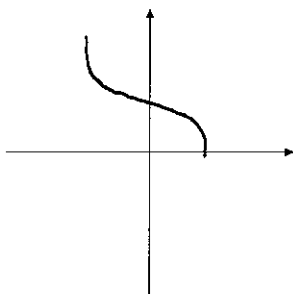Sketch the graphs. **For periodic functions, sketch at least 2 periods.** SCORE: _____ / 12 PTS
You only need to get the general position and shape correct. Do NOT plot points.
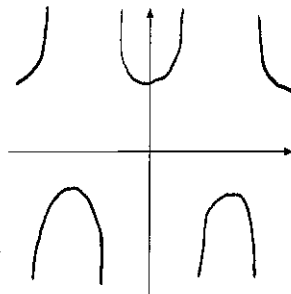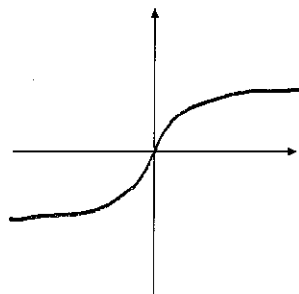
[a] $y = \cot x$ [b] $y = \cos^{-1} x$ [c] $y = \sec x$ [d] $y = \tan^{-1} x$

Fill in the blanks. Write "DNE" if the question has no answer. SCORE: _____ / 32 PTS

[a] $\arccos(-\frac{\sqrt{2}}{2}) = \underline{\frac{3\pi}{4}}$.

[b] $\tan^{-1}(-1) = \underline{-\frac{\pi}{4}}$.

[c] $\arctan \sqrt{3} = \underline{\frac{\pi}{3}}$.

[d] $\sin^{-1}\frac{1}{2} = \underline{\frac{\pi}{6}}$.

[e] $\tan(\tan^{-1} 2) = \underline{2}$.

[f] $\cos^{-1}(-\frac{1}{2})$ $\cos^{-1}(\cos\frac{4\pi}{3}) = \underline{\frac{2\pi}{3}}$.

[g] $\arccos(\cos\frac{3\pi}{4}) = \underline{\frac{3\pi}{4}}$.

[h] $\sin(\arcsin 2) = \underline{DNE}$.

[i] The range of $f(x) = \sec x$ is $\underline{(-\infty, -1] \cup [1, \infty)}$

[j] The domain of $f(x) = \arcsin x$ is $\underline{[-1, 1]}$.

[k] The equations of the asymptotes of $f(x) = \csc x$ are

$\underline{x = n\pi}$.

[l] The equations of the asymptotes of $f(x) = \tan^{-1} x$ are

$\underline{y = \pm\frac{\pi}{2}}$.

[m] The domain of $f(x) = \tan x$ is $\underline{x \neq \frac{\pi}{2} + n\pi}$

[n] The range of $f(x) = \cos^{-1} x$ is $\underline{[0, \pi]}$.

[o] As $x \to 0^-$, $\cot x \to \underline{-\infty}$.
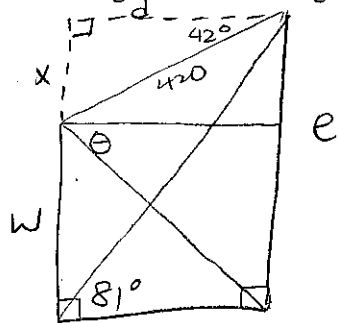
[p] As $x \to \frac{\pi}{2}^+$, $\sec x \to \underline{-\infty}$.

A 420 foot cable connects the roofs of two buildings.

From the base of the west building, the angle of elevation to the roof of the east building is $81°$.

From the roof of the east building, the angle of depression to the roof of the west building is $42°$.

[a]   Find the height of each building. (You may need to calculate other information that is not explicitly requested.)

$$\cos 42° = \frac{d}{420} \rightarrow d = 420 \cos 42° = 312$$

$$\sin 42° = \frac{x}{420} \rightarrow x = 420 \sin 42° = 281$$

$$\tan 81° = \frac{e}{312} \rightarrow e = 312 \tan 81° = 1970$$

$$w = 1970 - 281 = 1689$$

EAST BUILDING = 1970 ft

WEST BUILDING = 1689 ft

[b]   Find the angle of depression from the roof of the west building to the base of the east building.

$$\tan \theta = \frac{1689}{312} \rightarrow \theta = \tan^{-1} \frac{1689}{312} = 80°$$

BJ has unhealthy eating habits, which causes his weight to fluctuate up and down.

On Jan 21st (the day he had 3 midterms back-to-back), his weight reached a peak of 177.3 pounds.

On Feb 7th (the day he took a mental health day and slept for 18 hours), his weight reached a low of 168.1 pounds.

[a]   Assuming that BJ's weight corresponds to a sinusoidal function, find an equation for BJ's weight on the tth day after Jan 1st.

MIDDLE = $\frac{1}{2}(177.3 + 168.1) = 172.7$

AMPLITUDE = $\frac{1}{2}(177.3 - 168.1) = 4.6$

PERIOD = $2(37-20) = 34 = \frac{2\pi}{B} \rightarrow B = \frac{\pi}{17}$

JAN 21 $t = 37$
$t = 20$ FEB 7
+ cos

WEIGHT = $4.6 \cos \frac{\pi}{17}(t - 20) + 172.7$

[b]   Based on the answer of [a], how much will BJ weigh on Feb 22nd?

$$t = 52$$

$$4.6 \cos \frac{\pi}{17}(52 - 20) + 172.7 = 177 \text{ POUNDS}$$